UNIT 02

Forces & Motion

CHAPTER 02

Motion in a Straight Line

2.1 OBSERVING MOTION

People have been watching and recording things move for thousands of years. The motions of the heavens are some of the oldest recorded observations we have. Later, a need to measure the speed of advancing armies or athletes or ships required better ways of measuring distance and time. Over the centuries measurements became more accurate and now form the basis of modern physics. We can now measure distances and times to incredible accuracy.

Many types of motion are occurring around us all the time. Blood flow, moving bullets, cricket balls, athletics, cars, stars, planets, neutrinos and weaving looms are some of the areas where motion is measured. Some need to be measured carefully, others not. A car speedometer that is a few kilometres per hour over or under makes little difference but better accuracy is needed when timing a 100 metre sprint or controlling the speed of videotape through the heads of a VCR.

Sometimes the motion of objects doesn't make sense. Can you make sense of these questions?

- We live on a world that is round, yet we do not fall off. Many people used to believe the world was flat. Some still do. What evidence is there that it is round?
- Before Copernicus, most people believed that the Earth was stationary and the Sun moved around it. We now believe that the Earth is moving around the Sun but how do we know this?
- The Earth moves in a circular orbit and never slows down. Most objects in the world seem to travel in straight lines and slow down. Why is the Earth different?

The above three questions have several similarities. How many different things do they have in common?

Physics developed over the centuries as people pondered on these questions and came up with all sorts of different explanations. But people also found that knowing about the motion of everyday objects became more and more important.

It helps with your problem solving if you are familiar with some common motions and their measurements.

NEI Activity 2.1 SPEEDOMETER

Have a look at your family car's speedometer.

1 What is the maximum speed that it can record?

2 Do you know what your car's top speed is? If you don't, where would you find out? Assuming that it can't go as fast as the maximum value on the speedo, why do manufacturers use this sort in cars?

3 How many km/h are there per division?

4 The odometer (Greek *hodos* = 'a way') measures the total number of kilometres travelled by the car from when it was new. What is the maximum number of kilometres your car can travel before the odometer returns to all zeros?

Photo 2.1
A car speedometer.

5 Does your odometer measure to the nearest kilometre or tenth of a kilometre?
6 What is the maximum distance your 'trip meter' will record?
7 Some unscrupulous people illegally 'wind back' the odometer. What is the purpose of this and how do they do it?
8 Does the odometer go backwards when your car is reversed?
9 Does the speedo of your car go lower than zero when reversed?

Activity 2.2 SEWING MACHINE

Look at a sewing machine. How can you change the speed of a sewing machine motor? Is it variable? Are all electric motors controlled in the same way?

Activity 2.3 VIDEO RECORDER

If you have a VCR and can find the instruction manual, find out the tape speed on standard play. Should everyone in the class get the same result? Is the speed the same in videocameras? Are speed and image quality related?

A knowledge of physics enables us to analyse all types of motion. Without accurate measurement and control, life would be difficult indeed.

DISTANCE AND DISPLACEMENT 2.2

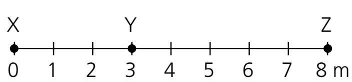
From the earliest times, being able to measure distances, angles and time was important in the daily lives of people. Often it was for religious reasons — worshipping sun gods; other times it was an attempt to plot the motion of the stars — a primitive astronomy. But sometimes it had a more practical purpose. Measuring distance, for instance, was important in the construction of houses, building canals and cultivating fields.

Plato told the story of how Posiedon (421 BC) inherited the island of Atlantis with its irrigated plain of 3000 by 2000 *stades* (a 'stade' is 185 metres, hence the word 'stadium'). Today, of course, we would be more likely to use metres or kilometres.

Whereas *length* is a measure of how long or wide an object is, we use the term *distance* to say how far the object has moved. A person travelling from one city to another may have moved a distance of 1200 km. In physics, we need to be able to measure not only distance but also 'displacement'.

Displacement is the change in position of an object in a given direction. You can think of it as the position measured relative to the origin. It is given the symbol 's'.

Figure 2.1

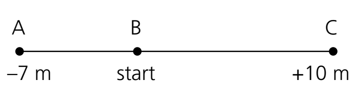


In Figure 2.1, if you started at point X and walked 8 m east to point Z and then turned around and walked 5 m west to point Y, you would have moved a distance of 13 m but would only have a displacement of 3 m east. That is, your position would only have changed by 3 m to the east. In symbols this could be written as s = 3 m E.

Displacement is called a **vector** quantity. That is, it involves both a number and a direction. Other vector quantities are velocity, acceleration and force. Quantities that do not include a direction are called **scalar** quantities. Distance, speed, mass and time are all scalar quantities. In the next chapter, vectors will be discussed in more detail.

When discussing vector quantities like displacement we use the compass points (N, E, W, S) to define directions as we did above, or alternatively, we can use a positive sign for forward motion or motion to the right and a negative sign for backward motion or motion to the left.

Figure 2.2



For example, in Figure 2.2 the displacement of C can be written as $s_C = +10$ m; and the displacement of A can be written as $s_A = -7$ m.

Either way, you'll need to be able to use both conventions. It's up to you and it is also up to you to define the positive and negative directions.

Representation of vector quantities

A vector quantity can be represented by a vector. A vector is an arrow. The length of the arrow represents the magnitude of the vector quantity, and the direction of the arrow shows the direction of the vector quantity. For example, the three vectors in Figure 2.3 represent cars travelling at 30 km/h east, 60 km/h west and 10 km/h north respectively.

Figure 2.3

When vectors do not lie along the compass points (N, E, S, W), angles need to be specified. Figure 2.4 shows how the direction is indicated.

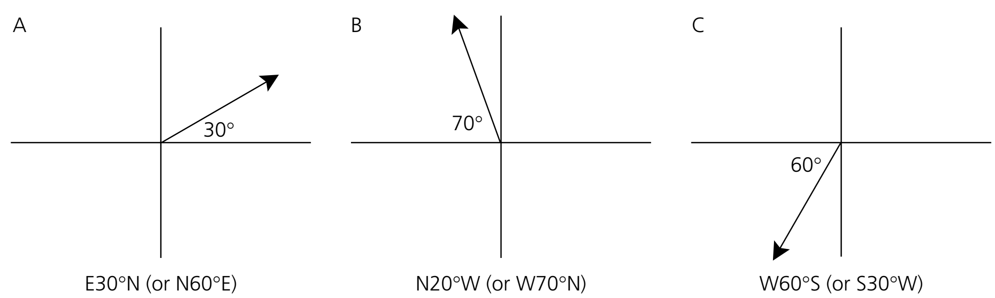


Figure 2.4

Students often find it hard to work out the directions. You can think of diagram A in Figure 2.4 as saying: going **e**ast but rotated **30°** to the **n**orth.

Example

In Figure 2.5, an orienteering competitor starts at point A and goes 2 km N, 4 km E and then 2 km S. What is the final displacement at point D?

Solution

The displacement at D is 4 km east (s_D = 4 km E).

Questions

1 In Figure 2.5:

(a) What is the displacement of the competitor at point B? (s_B = ?)
(b) What is the total distance travelled when at point D?
(c) What is the distance travelled when at point C?
(d) What is the displacement at point C? Remember to include the direction by stating the value of the angle CAD.

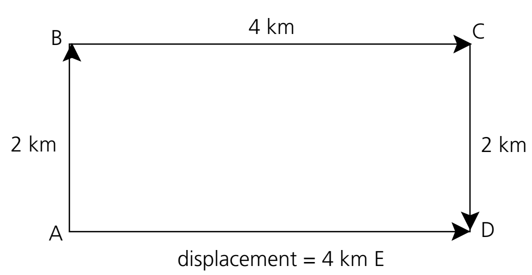


Figure 2.5
For question 1.

2 You watch your dog following a cat's scent trail. He walks 50 m north, turns and walks 60 m east and then walks 50 m south. What is his displacement?

3 A toy train is running around a circular track of diameter 120 cm. What is its distance travelled and its displacement after **(a)** one-half of a lap; **(b)** one full lap; **(c)** two laps; **(d)** one-quarter of a lap?

SPEED AND VELOCITY 2.3

NOVEL CHALLENGE

Try out these 'Fermi questions':

A How many golf balls will fit in a suitcase?

B How many hairs are there on a human head?

C How quickly does human hair grow (in kilometres per hour)?

D If all the people of the world were crowded together, how much area would we cover?

E What is the relative cost of fuel (per kilometre) of rickshaws and cars?

F How far does a car travel before a one-molecule layer of rubber is worn off the tyres?

Newspapers and magazines use the terms 'speed' and 'velocity' as if they mean the same thing. They do — almost. When a newspaper report mentions a high-speed car chase we know what is meant. But why do they also talk about hunting rifles being high-velocity? Newspapers say high-velocity atomic particles but they also talk of a cyclone's wind speed. Newspapers mean the same thing by speed and velocity. Why do you think they refer to some motions as speed and others as velocity?

In physics, speed and velocity are slightly different terms. Speed is a scalar quantity whereas velocity is a vector quantity. If it takes 2 hours to travel the 120 kilometres from Brisbane to Noosa then the average speed is 60 kilometres per hour. Speed is the rate at which distance is covered. Remember, the word 'rate' is a clue that something is being divided by time. Speed is always measured in terms of a unit of distance divided by a unit of time, such as metres per second.

$$\text{Average speed} = \frac{\text{total distance travelled}}{\text{time taken}}$$

This of course doesn't mean the driver sat on 60 km h^{-1} all the way. Sometimes the car would have gone at 100 km h^{-1} and at other times it would have been stationary. While the car's speedometer was reading 60 km h^{-1} then the car was actually travelling at that speed for that moment. This is called its instantaneous speed.

When we talk of a car's speed as being 60 km h^{-1} we have no idea about the direction it is travelling. Speed is a *scalar* quantity.

Velocity is defined as speed in a particular direction, for example 60 km h^{-1} north. Velocity is a *vector* quantity and the direction must be stated. In this book we represent a vector by printing its symbol in ***bold italics***.

Imagine a person running to catch a bus. Figure 2.6 shows him running north up Main Street at 5 m s^{-1}, turning east into Mary Street and continuing to run at 5 m s^{-1}. Although he was running at constant speed, his velocity changed because his direction changed.

Instantaneous velocity is similar to instantaneous speed except that a particular direction must be stated.

$$\text{Instantaneous velocity} = \frac{\text{small distance travelled in a stated direction}}{\text{time taken for this small distance}}$$

As distance moved in a stated direction is called 'displacement',

$$\text{instantaneous velocity} = \frac{\text{displacement}}{\text{time taken}}$$

$$\boldsymbol{v} = \frac{\boldsymbol{s}}{t} \text{ metres per second} = \frac{\text{metres}}{\text{seconds}}$$

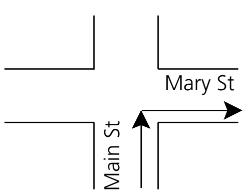


Figure 2.6
Turning into Mary Street. A change of direction means a change in velocity.

As with speed, we can use the term 'average velocity' to describe the motion of an object such as a car.

$$\text{Average velocity} = \frac{\text{displacement}}{\text{time taken}} \text{ or } \boldsymbol{v}_{av} = \frac{\Delta s}{\Delta t}$$

where Δ (delta, the Greek 'D') means 'change in', that is, Δt means change in time, but usually the deltas are omitted. The formula can be rearranged like this:

$$\boldsymbol{v}_{av} = \frac{s}{t} \qquad s = \boldsymbol{v}_{av}\, t \qquad t = \frac{s}{\boldsymbol{v}_{av}}$$

Table 2.1 COMPARISON OF SOME COMMON SPEEDS

MOVEMENT	m/s	km/h
Worm	0.005	0.01
Walking	1.4	5
Jogger	2.8	10
Cheetah	28	100
Sound in air	330	1200
Light	3×10^8	1 billion

Example 1

The trip meter on a car's speedo was set at zero and after a journey lasting half an hour the reading was 35 km. What was the average speed?

Solution

$$\text{Average speed} = \frac{\text{distance}}{\text{time}} = \frac{35\text{ km}}{0.5\text{ h}} = 70\text{ km/h or }70\text{ km h}^{-1}$$

Example 2

A person rides a bicycle 5 km east and then 5 km north (Figure 2.7). The trip takes 1.5 hours. Find **(a)** the total distance travelled; **(b)** the average speed; **(c)** the displacement; **(d)** the average velocity.

Solution

(a) Total distance = 5 km + 5 km = 10 km.

(b) Average speed $= \frac{\text{distance}}{\text{time}} = \frac{10\text{ km}}{1.5\text{ h}} = 6.7\text{ km h}^{-1}$.

(c) Displacement $= \sqrt{5^2 + 5^2} = 7$ km in a NE direction (s = 7 km NE or 7 km N45°E).

(d) Average velocity $= \frac{\text{displacement}}{\text{time}} = \frac{7\text{ km}}{1.5\text{ h}} = 4.7\text{ km h}^{-1}$ NE (N45°E).

NOVEL CHALLENGE

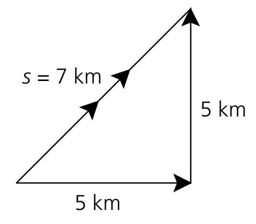
A lizard runs 30 m west, rests and heads 40 m north where it meets the base of a tree. It scampers 5 m straight up the tree. What is the magnitude of its displacement? How are you going to indicate the angle?

NOVEL CHALLENGE

The Greek symbol for 'D' is delta, Δ. In science, we use Δ to represent 'difference' because this also starts with 'D'. A delta is a triangular piece of flood plain where a river meets the sea, as in the Nile delta.

Was the symbol Δ called delta because it looked like the delta of a river, or was the flood plain called a delta because it looked like the Greek symbol Δ? The Greeks got the word delta from the inventors of the alphabet — the Phoenicians — who used it to mean 'door'.

Figure 2.7



Questions

4 To help you rearrange equations and substitute numbers, do the simple calculations shown in Table 2.2. Do not write in this book.

Table 2.2

DISPLACEMENT	TIME	VELOCITY
(a) 200 m	10 s	
(b) 50 km	1.5 h	
(c)	30 s	140 m/s
(d)	3 h	220 km/h
(e) 300 m		15 m/s
(f) 130 km		65 km/h

5 An archer can fire an arrow at 390 m/s. What time would an arrow take to hit a target 100 m away?

6 The highest speed on land in a car is 1190.4 km/h recorded by Stan Barrett (USA) in 1979 in his rocket-engined three-wheeled car at Edwards Airforce Base. What time would it have taken him to cover the 1.6 km test distance?

7 A person rides a bicycle to a shop by travelling 300 m north along a straight road and then travels west for another 400 m. If the trip takes 3 minutes, find **(a)** the average speed and **(b)** the average velocity.

8 A Ferrari Testarossa when driven by an experienced racing driver can cover 400 m from a standing start in 14.2 s. If it crosses the 400 m line at a speed of 203 km/h, what is its average speed?

NOVEL CHALLENGE

Confirm or refute the following statement: 'When an object is moved, its displacement can be smaller than the distance travelled, but the distance travelled can never be smaller than the displacement.'

DISPLACEMENT—TIME GRAPHS 2.4

It is often useful to show records of motion in the form of a graph. These can be in the form of a distance–time graph or as a displacement–time graph. In the graph shown in Figure 2.8 the position of a quarter-horse is shown, as recorded at six different times.

Photo 2.2
The Texas TI-83 graphing calculator and ranger has become a popular way of collecting and displaying data on the motion of objects, particularly in real-time.

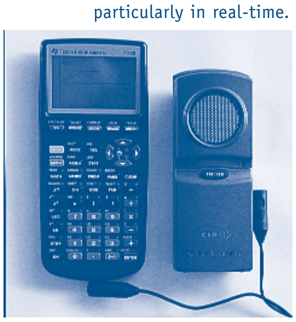


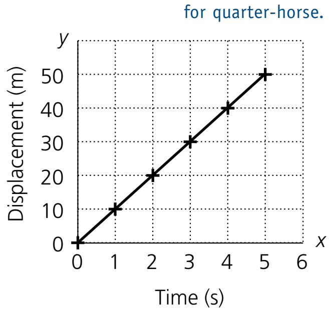
Table 2.3 DISPLACEMENT AND TIME MEASUREMENTS FOR A QUARTER-HORSE

Time elapsed (s)	0.0	1.0	2.0	3.0	4.0	5.0
Displacement (m)	0.0	10.0	20.0	30.0	40.0	50.0

In drawing the graph, it is usual to show the time elapsed on the *x*-axis and the displacement on the *y*-axis as in Figure 2.8.

Note that the six plotted points are the six observations of the quarter-horse. When we draw a line between these points we are assuming that the motion was uniform. This is called **interpolation** (Latin *inter* = 'between', *polire* = 'polish'; that is, to polish-up your data by supplying in-between points). When a line is extended past the first or last data points, this is called **extrapolation** (Latin *extra* = 'beyond').

Figure 2.8
Displacement–time graph for quarter-horse.



Questions

9 Table 2.4 records the motion of a dog chasing a ball. **(a)** Draw a displacement–time graph of the motion and describe it in words.

Table 2.4

Time elapsed (s)	0	1	2	3	4	5	6	7	8	9
Displacement (m)	0	2	4	4	4	6	6	4	2	0

(b) When was the dog stationary?
(c) When was its displacement increasing?
(d) When was the dog moving with the greatest speed?

2.5 SLOPE AND VELOCITY

Figure 2.9 is the displacement–time graph of a sprinter who runs 100 m in 10 s, rests for 20 s and then sprints back to the starting point in the next 30 s.

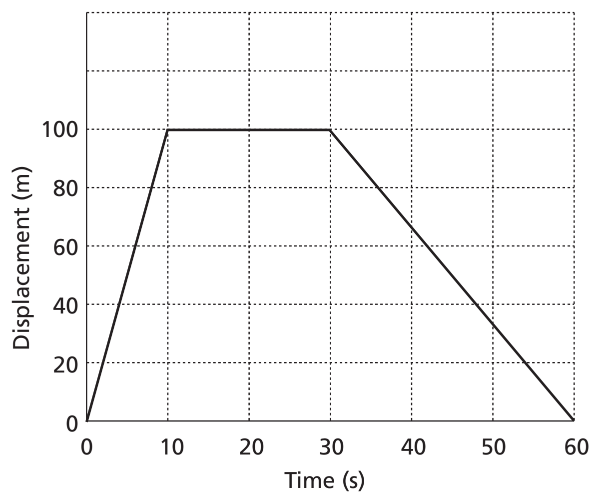


Figure 2.9
Displacement–time graph of sprinter.

The sprinter's average velocity in the first 10 seconds is calculated by dividing the displacement by the time taken. This is the same as calculating the slope of the line. The slope of any line is given by change in y divided by change in x ('rise over run'):

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

From the above displacement–time graph, the slope is given by:

$$\boldsymbol{v}_{av} = \text{slope} = \frac{\Delta y}{\Delta x} = \frac{\text{change in position}}{\text{time taken}} = \frac{100\text{ m} - 0\text{ m}}{10\text{ s} - 0\text{ s}} = 10\text{ m s}^{-1}$$

The slope of the line is constant for the first 10 seconds, indicating that the velocity was also constant.

From $t = 30$ s to $t = 60$ s the average velocity can be calculated:

$$\boldsymbol{v}_{av} = \frac{0 - 100}{60 - 30} = 3.3\text{ m s}^{-1}.$$

Note that when the slope of the line is positive the velocity is in the positive direction. When the slope is negative the velocity is negative; this simply means that the direction of motion has reversed.

Questions

10 For the graph of a rollerskater shown in Figure 2.10:
(a) calculate his average velocity for each of the five sections of the graph;
(b) calculate his average velocity for the whole journey;
(c) calculate his average speed for the whole journey.

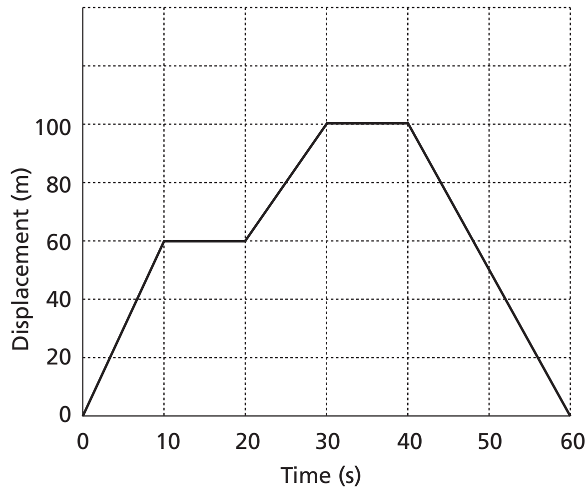


Figure 2.10
Displacement–time graph of rollerskater.

ACCELERATION 2.6

The velocity of a car increases when it starts moving from rest and decreases when the brakes are applied and it slows down. Cars can thus accelerate (speed up) or decelerate (slow down). The rate at which the velocity changes is called its **acceleration**. Consider the measurements of a car taking off from the traffic lights, shown in Table 2.5.

NOVEL CHALLENGE

The 08.00 express from Cleveland to Brisbane arrives at 9.00, and the 08.30 from Brisbane to Cleveland arrives at 9.30. Assuming both trains travel at constant speed, at what time should they pass each other?

Table 2.5

TIME ELASPSED (s)	DISPLACEMENT (m)	VELOCITY (m s^{-1})
0	0	0
1	1	2
2	4	4
3	9	6
4	16	8
5	25	10

The car's velocity is changing by 2 m s^{-1} every second. Its acceleration is said to be 2 m s^{-1} per second or 2 m s^{-2}. The formula for acceleration is then:

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time taken}} = \frac{\Delta \boldsymbol{v}}{\Delta t}$$

$$= \frac{\text{final velocity} - \text{initial velocity}}{\text{time taken}}$$

$$\boldsymbol{a} = \frac{\boldsymbol{v} - \boldsymbol{u}}{t}$$

where $\boldsymbol{v}$ is the final velocity and $\boldsymbol{u}$ is the initial velocity.

Note that the *change* of displacement is increasing for every second elapsed. In the 1st second, the displacement changes by 1 m, whereas in the 2nd second the displacement changes by 3 m. The above data are plotted on the three graphs shown in Figure 2.11. Graphs of uniformly accelerated motion are related as shown in the figure.

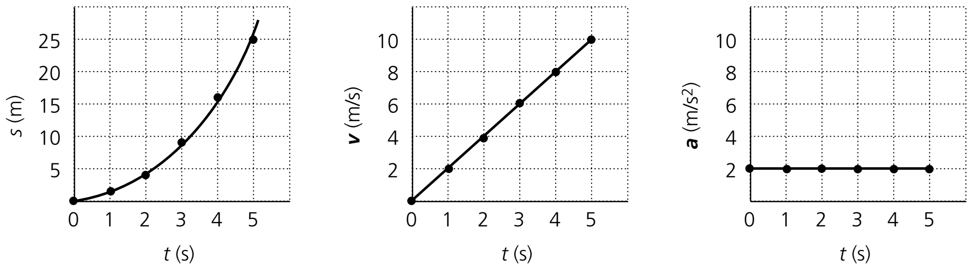


Figure 2.11

Questions

11 Plot an **s**–*t* graph and a ***v***–*t* graph of the data of a ball rolling down an incline, listed in Table 2.6. Don't write in this book.

Table 2.6

TIME ELASPSED (s)	DISPLACEMENT (m)	VELOCITY (m s^{-1})
0	0	0
1	3	6
2	12	12
3	27	18
4	48	24
5	75	30

By inspection of the data, state the acceleration of the rolling ball.

Example

American experiments reveal that the beak of the red-headed woodpecker hits the bark of a tree at an impact velocity of 5.8 m s^{-1} and comes to rest in 0.059 s. Calculate the deceleration of the bird's head.

Solution

$$\boldsymbol{a} = \frac{\boldsymbol{v} - \boldsymbol{u}}{t} = \frac{0 - 5.8}{0.059} = -98 \text{ m s}^{-2} \ (-9.8 \times 10^{1} \text{ m s}^{-2})$$

The negative sign indicates that the bird slowed down.

Questions

12 Complete Table 2.7. This will give you practice at manipulating the equation for acceleration. Don't write in this book.

Table 2.7

	v (m s^{-1})	u (m s^{-1})	Δv (m s^{-1})	t (s)	a (m s^{-1})
(a)	18	10		2.0	
(b)	42		4		4.0
(c)		20		10	−2.0
(d)	18	25		3.5	
(e)		−5		1.3	−0.5

NOVEL CHALLENGE

Here's an interesting theory that could be investigated experimentally. R. McNeill Alexander from Leeds University, England, measured the speed at which animals switched from walking to running. For humans, the speed is about 8 km h^{-1}. He developed a rule which, stated mathematically, is: $\boldsymbol{v}^2 = \frac{1}{2}\, g\, d_H$, where **v** is the speed at which an animal switches, d_H is the distance from the hip to the ground, and $\boldsymbol{g}$ is the acceleration due to gravity. His rule applies to animals from insects to humans. Can you confirm this rule by experiment?

13 The highest road-tested acceleration reported for a standard production car is 0 to 96.5 km/h (26.8 m s^{-1}) in 3.98 s for a Ferrari F40 driven by Mark Hales of *Fast Lane Magazine* in the UK on 9 February 1989. Calculate the acceleration of the car.

14 The highest speed by a rocket-engined wheeled land vehicle was 1046 km h^{-1} (290 m s^{-1}) recorded by Gary Gabelich in The Blue Flame on the Bonneville Salt Flats in 1970. His acceleration was measured as 4.2 m s^{-2} in getting to this speed from rest. How many seconds would he have taken to reach this speed?

15 The head of a rattlesnake can accelerate at 50 m s^{-2} when striking a victim. If a car could do as well, how long would it take for it to reach a speed of 27 m s^{-1} (100 km h^{-1}) from rest?

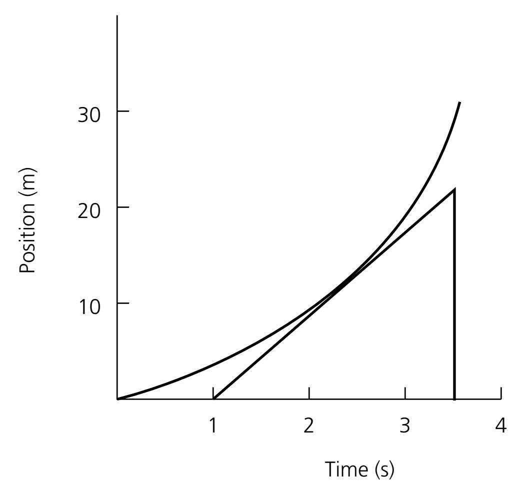
16 A muon (an elementary particle) enters an electric field with a speed of 5.00×10^6 m s^{-1}, whereupon the field causes it to decelerate at 1.25×10^{14} m s^{-2}. How much time elapses before it stops?

INSTANTANEOUS VELOCITY 2.7

When you read a car's speedo you are seeing the instantaneous speed of the car. If it reads 60 km h^{-1}, then it means that at the current speed you would cover 60 km in 1 hour. But you could be accelerating and the speedo might be gradually changing from 50 km h^{-1} to 100 km h^{-1}. When it read 60 km h^{-1} this was its **instantaneous speed**. If a direction is also specified, then you would be talking about its **instantaneous velocity**.

Consider the case of an accelerating car. In this case the velocity is getting faster as time goes by so the s–t graph is a curve as shown in Figure 2.12.

Figure 2.12(a)
The instantaneous velocity at time t = 2.5 s is given by the slope of the tangent to the curve at that point.



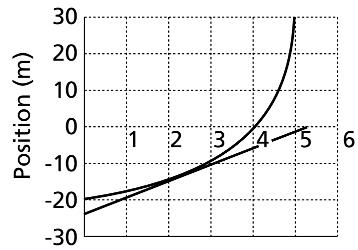
To calculate the instantaneous velocity at 2.5 s in Figure 2.12**(a)**, a tangent is drawn to the curve at the 2.5 s mark. The tangent is a line that just touches the curve at that point. The slope of the tangent can be calculated:

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{20 - 0}{3.5 - 1} = 8 \text{ m s}^{-1}$$

A more difficult case is shown in Figure 2.12**(b)**. To calculate the instantaneous velocity at 2 seconds in Figure 2.12**(b)**, a tangent to the curve has been drawn and the slope of the tangent can be calculated:

$$\text{Slope} = \frac{0 - -23}{5.2} = 4.4 \text{ m s}^{-1}$$

Figure 2.12(b)



Question

17 From Figure 2.12b: **(a)** Calculate the instantaneous velocity at 4 s. **(b)** Calculate the average velocity over the whole 5 s. Do not draw in this book. Use your ruler.

2.8 VELOCITY–TIME GRAPHS

Graphs can also be used to show the changes in velocity of an object with time. The graph in Figure 2.13 represents a car being accelerated from rest to 20 m s^{-1} in 10 s and being held at that speed for 10 s before the driver slows down to a stop.

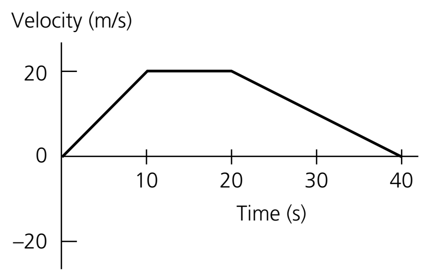


Figure 2.13

A straight line sloping upward indicates constant acceleration, whereas a straight line sloping down indicates deceleration or negative acceleration. A horizontal line indicates zero acceleration, that is, constant velocity. As the formula for acceleration $\boldsymbol{a} = \dfrac{\boldsymbol{v} - \boldsymbol{u}}{t}$ is equivalent to $\dfrac{\Delta y}{\Delta x}$ then acceleration is equal to the slope of a *v*–*t* graph.

The displacement can be calculated by finding the area under the line. For instance, in the case above, the car has travelled at an average speed of 10 m s^{-1} for the first 10 s. Hence the displacement must be 10 m s^{-1} × 10 s = 100 m. The area under the line for the first 10 s is (20 × 10)/2, that is, (base × height)/2, which equals 100 m also. **The area under a *v*–*t* graph equals displacement.**

Example

Using the graph shown in Figure 2.13:

(a) Calculate the acceleration of the car at **(i)** 5 s, **(ii)** 15 s and **(iii)** 30 s.
(b) Calculate the displacement after 40 s.
(c) Calculate the average velocity.
(b) Sketch an acceleration–time graph.

Solution

(a) (i) The acceleration at 5 s is equal to the slope at 5 s:

$$\boldsymbol{a} = \text{slope} = \frac{20 - 0}{10} = 2 \text{ m s}^{-2}$$

(ii) Slope equals zero, therefore acceleration is zero.

(iii) Slope $= \dfrac{0 - 20}{40 - 20} = -1 \text{ m s}^{-2}$.

NOVEL CHALLENGE

You have learnt that the rate of change of position with respect to time is **velocity**, and the rate of change of velocity is **acceleration**. Did you know that the rate of change of acceleration is known as **jerk** (symbol ***j***)? Jerk is important when evaluating the destructive effect of motion on a mechanism, or the discomfort caused to passengers in a vehicle. The movement of delicate instruments needs to be kept within specified limits of jerk as well as acceleration to avoid damage. When designing a train the engineers will typically be required to keep the jerk less than 2 m s^{-3} for passenger comfort. In the aerospace industry they even have such a thing as a jerkmeter — an instrument for measuring jerk. In the case of the Hubble space telescope, the engineers specified limits on the magnitude of the rate of change of jerk. There is no universally accepted name for this fourth derivative. Is the slope of, or area under, an *a*–*t* graph related to jerk? Does the slope of, or area under, a jerk–time graph mean anything?

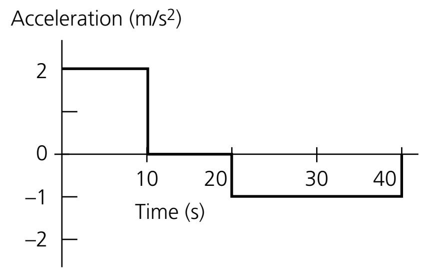
(b) Total area = $\dfrac{20 \times 10}{2} + 20 \times 10 + \dfrac{20 \times 20}{2}$ = 500 m (or 5×10^2 m).

(c) Average velocity = displacement ÷ time:

$$v_{av} = \frac{s}{t} = \frac{500}{40} = 12.5 \text{ m s}^{-1} \text{ (or 10 m s}^{-1} \text{ to one significant figure)}$$

(d) See Figure 2.14.

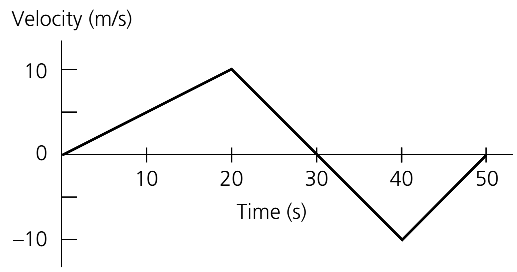
Figure 2.14



For cases where the velocity becomes negative, the area beneath the *x*-axis is also negative and this must be taken into account when calculating displacement.

For example, imagine the motion of a bungee jumper, jumping off a tower (Figure 2.15).

Figure 2.15



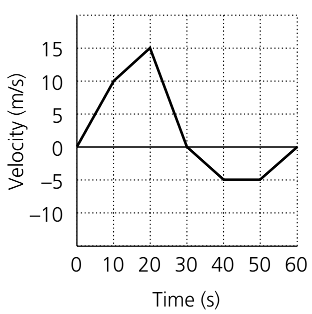
The **displacement** after 50 s is $\dfrac{30 \times 10}{2} + \dfrac{20 \times -10}{2}$ = 50 m. The **distance** travelled, however, is not a vector quantity and the area underneath the *x*-axis is not considered to be negative. The distance travelled is 250 m (150 m down plus 100 m back up).

Questions

18 For the motion of the bungee jumper shown in Figure 2.15 above:
- **(a)** calculate the displacement and distance travelled after 40 s;
- **(b)** calculate the acceleration at 10 s, 30 s and 45 s;
- **(c)** sketch an acceleration–time graph.
- **(d)** When was he stationary?
- **(e)** When was his acceleration constant but not zero?
- **(f)** When was his velocity constant but not zero?

19 The graph shown in Figure 2.16 illustrates the motion of a skateboard rider.
- **(a)** Calculate his displacement after 1 minute.
- **(b)** Calculate how far he travelled in the minute.

Figure 2.16
For question 19.



(c) At what stage was the magnitude of his acceleration the greatest?
(d) When was he stationary?
(e) When was his acceleration negative and constant?
(f) When was his velocity constant but not zero?

20 Olympic equestrian 'Three-day eventing' is held over 4 days. The first 2 days consist of dressage while the 4th day is for show-jumping. The 3rd day is the speed and endurance section. At the 1996 Atlanta Olympics, the gold medallist achieved these results for Day 3:
Stage 1 (The Trot) was at 13 km h^{-1} for 10 minutes followed by Stage 2 (The Fast Steeplechase), which took 5 minutes at 41 km h^{-1}. Stage 3 was another trot the same as Stage 1. Before Stage 4 there was a compulsory 10 minute rest. Stage 4 was a testing 14-minute cross-country gallop at 34 km h^{-1}.
(a) Draw a $\boldsymbol{v}$–t graph of the motion.
(b) Calculate the total distance travelled in this event.

2.9 EQUATIONS OF MOTION

The equations used so far can be combined to provide other useful ways of calculating and describing the motion of objects.

In real life we encounter several main kinds of motion:

- Constant velocity (zero acceleration).
- Regularly changing velocity (constant acceleration).

Case 1: Constant velocity

The simplest kind of motion we can study is that in which the object moves with constant velocity and hence zero acceleration. The graphs for this type of motion are illustrated in Figure 2.17. Some examples drawn from everyday life are:

- a car being driven at 60 km h^{-1}
- ball bearings being rolled on a very smooth horizontal surface
- a person jogging
- water flowing in a pipe.

Accelerated motion is also easy to find. Examples are:

- objects falling freely under gravity
- a car moving away from the traffic lights
- an aircraft being catapulted by a steam catapult from an aircraft carrier.

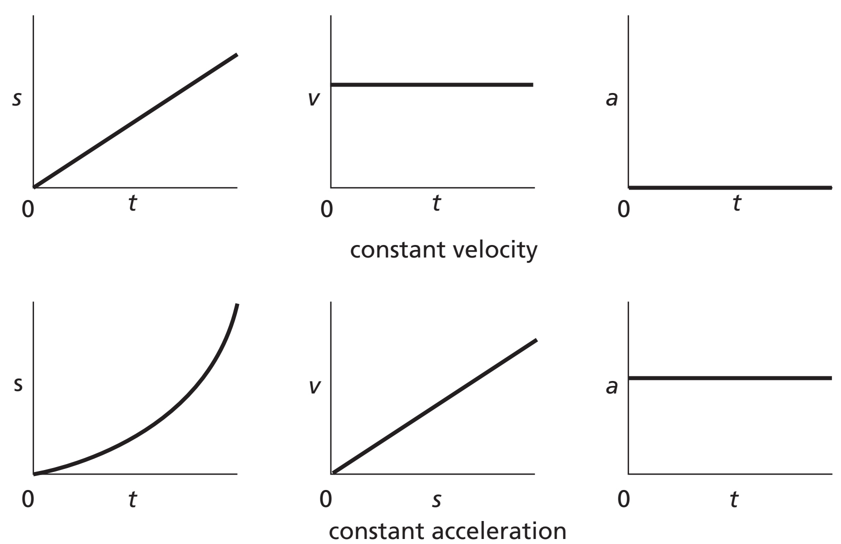
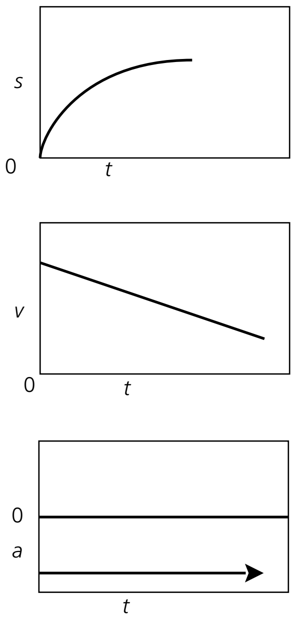


Figure 2.17
Motion graphs showing corresponding displacement–time, velocity–time and acceleration–time relations for situations of constant velocity and constant acceleration.

Figure 2.18



Case 2: Constant (uniform) acceleration

Graphs representing this type of motion are also shown in Figure 2.17. Objects falling freely under gravity are the most common examples of this.

Another case of constant acceleration is for an object slowing down (decelerating or negative acceleration). Figure 2.18 shows graphs of this motion.

The quantities displacement, time, velocity and acceleration are all related to each other. In this book the symbols shown in Table 2.8 will be used.

Table 2.8

QUANTITY	QUANTITY SYMBOL	UNIT	UNIT SYMBOL
Displacement	$\boldsymbol{s}$	metres	m
Initial velocity	$\boldsymbol{u}$	metres per second	m s^{-1}
Final velocity	$\mathbf{v}$	metres per second	m s^{-1}
Acceleration	$\mathbf{a}$	metres per second per second	m s^{-2}
Time (elapsed)	t	second	s

Development of formulas

1 Acceleration = $\dfrac{\text{final velocity} - \text{initial velocity}}{\text{time}}$:

$$\boldsymbol{a} = \frac{\boldsymbol{v} - \boldsymbol{u}}{t} \quad \text{or} \quad \boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t \qquad (1)$$

2 Average velocity = $\dfrac{\boldsymbol{s}}{t}$ and also equals $\dfrac{\boldsymbol{u} + \boldsymbol{v}}{2}$

$$\frac{\boldsymbol{u} + \boldsymbol{v}}{2} = \frac{\boldsymbol{s}}{t} \quad \text{or} \quad \boldsymbol{s} = \frac{(\boldsymbol{u} + \boldsymbol{v})t}{2} \qquad (2)$$

3 If we substitute equation (1) into (2) we get:

$$\boldsymbol{s} = \frac{(\boldsymbol{u} + (\boldsymbol{u} + \boldsymbol{a}t))t}{2} \quad \text{or} \quad \boldsymbol{s} = \boldsymbol{u}t + \tfrac{1}{2}\boldsymbol{a}t^2 \qquad (3)$$

4 From equation (1) we get $t = \dfrac{\boldsymbol{v} - \boldsymbol{u}}{\boldsymbol{a}}$. Substituting this into equation (2), we get:

$$\boldsymbol{s} = \frac{\boldsymbol{u} + \boldsymbol{v}}{2} \times \frac{\boldsymbol{v} - \boldsymbol{u}}{\boldsymbol{a}} \quad \text{or} \quad 2\boldsymbol{a}\boldsymbol{s} = (\boldsymbol{u} + \boldsymbol{v})(\boldsymbol{v} - \boldsymbol{u})$$

$$2\boldsymbol{a}\boldsymbol{s} = \boldsymbol{v}^2 - \boldsymbol{u}^2$$

$$\boldsymbol{v}^2 = \boldsymbol{u}^2 + 2\boldsymbol{a}\boldsymbol{s} \qquad (4)$$

Note: these formulas only apply when the acceleration is constant and the motion is in a straight line. Velocity, acceleration and displacement are vector quantities and therefore may be positive or negative.

We can finally summarise the equations of motion as listed in Table 2.9.

NOVEL CHALLENGE

A car travels from A to B at an average speed of 100 km/h and returns at 60 km/h. *What is the average speed for the journey?*

Table 2.9 SUMMARY OF FORMULAS

MOTION WITH UNIFORM VELOCITY	MOTION WITH UNIFORM ACCELERATION
v = constant	a = constant
$v_{av} = \frac{s}{t}$	$v_{av} = \frac{v + u}{2}$
$s = vt$	$v = u + at$
	$s = ut + \frac{1}{2}at^2$
	$v^2 = u^2 + 2as$
	$s = \frac{(u + v)t}{2}$

Example 1

A car starts from rest and reaches a velocity of 60 km h^{-1} (16.67 m s^{-1}) in 8 seconds. Assuming the acceleration to be constant, calculate **(a)** the acceleration and **(b)** the displacement in this time interval.

Solution

Data: $u = 0$, $v = 16.67$ m s^{-1}, $t = 8$ s, $a = ?$, $s = ?$

(a) $$a = \frac{v - u}{t} = \frac{16.67 - 0}{8} = 2.08 \text{ m s}^{-2}.$$

(b) $$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 2.08 \times 8^2 = 66.6 \text{ m}.$$

Example 2

A train starting from rest travels 30 m in 6 s. Find **(a)** its acceleration and **(b)** its velocity after the 6 s.

Solution

Data: $u = 0$, $s = 30$ m, $t = 6$ s, $a = ?$, $v = ?$

(a)
$$s = ut + \frac{1}{2}at^2$$
$$30 = 0 + \frac{1}{2}a6^2$$
$$30 = 18a$$
$$a = 1.67 \text{ m s}^{-2}.$$

(b)
$$v = u + at$$
$$= 0 + 1.67 \times 6$$
$$= 10 \text{ m s}^{-1}.$$

Questions

21 Table 2.10 will give you practice in selecting equations of motion and substituting values into them. Complete the table but do not write in this book.

NOVEL CHALLENGE

A column of troops 3 km long is marching along a road. An officer rides from the rear to the head of the column and back once, and he reaches the rear of the column just as an advance of 4 km has been made from where he first left.
How far did he ride?

NOVEL CHALLENGE

A man goes from A to B at 30 km/h.
How fast must he return to average 60 km/h for the whole trip?

NOVEL CHALLENGE

A boy is carried up an escalator in 1 minute. He can walk up a stationary escalator in 3 minutes. *How long will it take him to walk up a moving escalator?*

Table 2.10

QUESTION	*s* (m)	*u* ($m\ s^{-1}$)	*v* ($m\ s^{-1}$)	*a* ($m\ s^{-1}$)	*t* (s)
(a)		0		2.5	3
(b)	100	0			2.4
(c)		10	25	2	
(d)	300	9		1.5	
(e)	40	2		4	
(f)		10	5		2.5
(g)	160	50			8

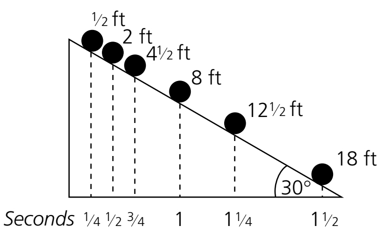
22 A cyclist starts from rest and attains a velocity of 21 m s^{-1} in 3.5 seconds. Calculate **(a)** the acceleration, assumed constant; **(b)** the displacement.

23 A bus travelling in a straight line accelerates from 60 km h^{-1} to 100 km h^{-1} in 1 minute. Calculate the acceleration in m s^{-2}.

24 The click beetle (*Athous haemorrhoidalis*) experiences an acceleration of 24 000 m s^{-2} over a distance of 5 mm when it jack-knifes into the air to avoid predators. For what time duration does this acceleration occur?

ACCELERATION DUE TO GRAVITY 2.10

Figure 2.19
Galileo's data from his inclined plane experiments.



One of the most common examples of motion in a straight line with uniform acceleration is that of an object that falls freely due to gravity. Until Galileo (1564–1642), people thought that heavy objects fell faster than light objects. They saw no need for experiments that may have confirmed or refuted these beliefs. They relied on the theories of Aristotle, who believed that objects fell at speeds that depended on their weight. Galileo performed some of the earliest experiments, which showed that both heavy and light objects in the absence of air and other resistance fell with constant acceleration. Thus, two objects of different masses, dropped from the same height at the same time, should strike the ground simultaneously.

Motion due to gravity can take two main forms. The first is vertical motion, where the object moves in one dimension only, that is, up and down. The second is projectile motion, where the object moves horizontally as well as vertically, for example a stone thrown off a cliff. Only vertical motion will be dealt with in this chapter. Projectile motion will be discussed in Chapter 5.

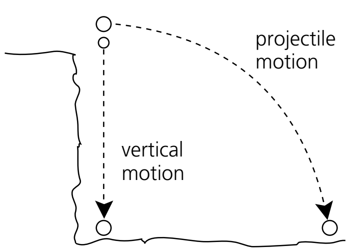
Types of free-fall motion

Free-fall motion can be grouped into two classes:

1. The object is being dropped or thrown down.
2. The object is being thrown upward.

Positive and negative convention When dealing with calculations involving acceleration due to gravity we need to assign a positive and negative direction of motion. In this chapter we will use the convention in which **up is positive**. Throughout the world, this is the most common. It is a matter of your choice, however, but you may find it simplest to stay with the one convention.

Figure 2.20
Vertical and projectile motion.



Acceleration due to gravity is constant at 10 m s^{-2} in the negative direction (down), hence $a = -10$ m s^{-2}. This means that an object will increase its velocity in the negative direction by 10 m s^{-1} every second, or by 10 metres per second *per second*. Students often think that a negative acceleration means *de*celeration or slowing down but this is not always so. If an object is moving in the negative direction (down) and has negative acceleration then it will get faster in that negative direction. If it is moving in the positive direction (upward) and has a negative acceleration then it is slowing down in that positive direction.

Case 1: Dropped or thrown down

Most typically, these situations involve dropping a rock off a cliff or throwing something vertically downward. In both cases the velocity increases. The only difference is the initial velocity. When dropped, the initial velocity is zero but when thrown down the velocity begins at some negative value. Either way, the velocity increases in the negative direction.

Example 1

A spanner is dropped from a sixth-floor window and takes 2.2 s to hit the ground. Calculate **(a)** the height from which it was dropped and **(b)** its impact velocity.

Solution

Take the downward direction as negative.

Data: $\boldsymbol{u} = 0 \text{ m s}^{-1}$; $\boldsymbol{a} = -10 \text{ m s}^{-2}$; $t = 2.2$ s; $\mathbf{s}$ = ?; $\boldsymbol{v}$ = ?

(a)

$$\begin{aligned} \mathbf{s} &= \boldsymbol{u}t + \tfrac{1}{2}\boldsymbol{a}t^2 \\ &= 0 + \tfrac{1}{2} \times -10 \times 2.2^2 \\ &= -24.2 \text{ m} \end{aligned}$$

(b)

$$\begin{aligned} \boldsymbol{v} &= \boldsymbol{u} + \boldsymbol{a}t \\ &= 0 + -10 \times 2.2 \\ &= -22 \text{ m s}^{-1} \end{aligned}$$

NOVEL CHALLENGE

If you put a row of coins on a 1 metre ruler that has one end on the ground and let the other end fall, which coins will stay on the ruler and which ones will be left behind?

Example 2

The Zero Gravity Research Facility at the NASA Research Centre includes a 150 m drop tower. This is an evacuated vertical tower through which a 1 m diameter sphere can be dropped. If this sphere is projected downward at an initial speed of 5 m s^{-1}, how long would it take to reach the bottom?

Solution

Data: $\mathbf{s} = -150$ m; $\boldsymbol{u} = -5 \text{ m s}^{-1}$; $\boldsymbol{a} = -10 \text{ m s}^{-1}$; t = ?

$$\begin{aligned} \mathbf{s} &= \boldsymbol{u}t + \tfrac{1}{2}\boldsymbol{a}t^2 \\ -150 &= -5 \times t + \tfrac{1}{2} \times -10 \times t^2 \\ 5t^2 + 5t - 150 &= 0 \\ t^2 + t - 30 &= 0 \\ (t - 5)\,(t + 6) &= 0 \end{aligned}$$

Hence $t = -6$ s or $t = +5$ s. The answer must be 5 s as the negative time is not meaningful here.

Note: in cases where the quadratic equation doesn't factorise simply as shown above, the quadratic formula will be needed:

$$\text{Quadratic formula: } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Without the quadratic formula you would first need to calculate $\boldsymbol{v}$.

PHYSICS FACT
THE POWER OF MINUS

(a) In 1962, the Mariner I mission launched towards Venus but the rocket separated from the boosters too soon and plunged into the ocean 4 minutes after take-off. Some klutz left a negative (–) sign out of the computer program.

(b) The old equation for the energy of a photon was $\frac{1}{2}mv^2 = hf$. Einstein added $-W$ and got a Nobel Prize.

Example 3

A person aboard a balloon moving downward at 30 m s^{-1} drops a sandbag at an elevation of 500 m. **(a)** What time will it take for the sandbag to hit the ground? **(b)** What will be the speed of the bag on impact?

Solution

Data: $\boldsymbol{s} = -500$ m; $\boldsymbol{u} = -30$ m s^{-1}; $\boldsymbol{a} = -10$ m s^{-2}; $t = ?$

(a)

$$\boldsymbol{s} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$$

$$-500 = -30t + \frac{1}{2} \times -10 \times t^2$$

$$t^2 + 6t - 100 = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4\boldsymbol{a}c}}{2\boldsymbol{a}} = \frac{-6 \pm \sqrt{6^2 - 4 \times 1 \times -100}}{2 \times 1}$$

$$= \frac{-6 \pm 20.9}{2}$$

$$= -13.4 \text{ s or } +7.4 \text{ s}$$

The negative time has no real meaning in this case, so the answer is 7.4 s.

(b)

$$\begin{aligned} \boldsymbol{v} &= \boldsymbol{u} + \boldsymbol{a}t \\ &= -30 + -10 \times 7.4 \\ &= -30 + -74 \\ &= -104 \text{ m s}^{-1} \end{aligned}$$

NOVEL CHALLENGE

A flea (*Pulex irritans*) can jump about 4 m high. If the flea was a big as a person, how high would it be able to jump (proportionally)?

Remember, the magnitude of the acceleration due to gravity ($\boldsymbol{g}$) is about 9.8 m s^{-2}. This means that an object falling freely under gravity increases its speed by about 10 m s^{-1} every second. It is given the negative sign because we have adopted the convention that upward is positive and downward is negative.

SR Activity 2.4 VERTICAL MOTION ON THE SPREADSHEET

If you have access to a computer and are familiar with spreadsheeting, set up a spreadsheet with the following headings (Table 2.11):

Table 2.11 SPREADSHEET

	A	B	C
1	t (s)	s (m)	**v** (m s^{-1})
2	0	0	0
3	1		
4	2		

1 The formula for cell B2 would be = (0.5 * 9.8*A2*A2) for example.

2 Extend Column A to 20 seconds and compute the value for all cells.

3 Use the graph commands to draw s–t and $\boldsymbol{v}$–t graphs. Are they what you would expect?

4 Discuss your output.

Case 2: Throwing an object upward

When a ball is thrown vertically upward, it starts at a high initial velocity in the positive direction, gradually slows to a halt at the top of its flight and gradually increases velocity in the negative direction until it returns to the ground.

Figure 2.21 shows the flight of the ball; although its downward path is exactly the same as the upward path, it is drawn slightly to the right for clarity.

Note: it can be shown that:

- velocity equals zero at the top of flight
- time of flight up equals time down
- acceleration is constant even at the top of flight when velocity is zero
- initial speed equals final speed
- final velocity equals the negative of the initial velocity
- air resistance is negligible and can be neglected.

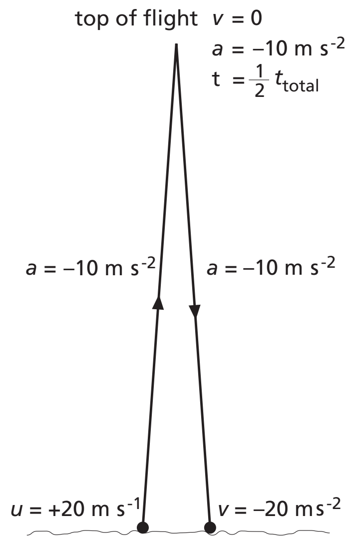


Figure 2.21
Trajectory of an object thrown vertically.

Example

A ball is thrown vertically upward at 20 m s^{-1}. Ignoring air resistance and taking $g = -10$ m s^{-2}, calculate **(a)** how high it goes; **(b)** the time taken to reach this height; **(c)** the time taken to reach the ground from the highest point; **(d)** the final velocity; **(e)** time of flight.

Solution

Data: Taking down as negative: $\boldsymbol{u} = +20$ m s^{-1}, $\boldsymbol{a} = -10$ m s^{-2}, $\boldsymbol{s} = 0$ m.

(a) At the top of flight $\boldsymbol{v} = 0$ m s^{-1}:

$$\boldsymbol{v}^2 = \boldsymbol{u}^2 + 2\boldsymbol{as}$$
$$0 = (+20)^2 + 2 \times -10 \times \boldsymbol{s}$$
$$20\boldsymbol{s} = 400$$
$$\boldsymbol{s} = 20 \text{ m (i.e. 20 m up in the air).}$$

(b)

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$$
$$0 = +20 + -10t$$
$$t = 2 \text{ s}$$

(c) The ground is 20 m in the negative direction from the top of flight. Hence $s = -20$ m:

$$\boldsymbol{s} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$$
$$-20 = 0 + -5t^2$$
$$t = 2 \text{ s}$$

(d)

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$$
$$= 0 + -10 \times 2$$
$$= -20 \text{ m s}^{-1}$$

(e) Time up = 2 s; time down = 2 s. Hence total time of flight equals 4 s. Using the equations of motion it can be shown that when the displacement is zero, the time for this to occur is zero seconds (the start) and 4 seconds (the finish):

$$\boldsymbol{s} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$$
$$0 = +20t + \frac{1}{2} \times -10t^2$$
$$5t^2 = 20t$$
$$t = 4 \text{ s}$$

Questions

25 A rock is dropped off a cliff and it takes 4 s to reach the base below. How high is the cliff?

26 A pot-plant falls 25 m from rest to the ground below.
(a) What is its impact velocity?
(b) What time did it take to fall?

27 A rock is launched vertically upward from the ground at a starting speed of 35 m s^{-1}.
(a) What is the maximum height reached?
(b) What time does it take to reach this maximum height?
(c) What time does it take to fall back to the ground again?

28 A person in a balloon moving vertically upward at a constant speed of 4.9 m s^{-1} drops a sandbag at an elevation of 98 m.
(a) What time will it take until the sandbag hits the ground?
(b) What will be the velocity of the sandbag on impact?

29 A startled armadillo leaps upward and rises 54.4 cm in 0.20 s and keeps rising.
(a) What was its initial speed? **(b)** What is its speed at this height?
(c) How much higher does it go?

GRAPHS OF FREE-FALL MOTION 2.11

The two most common types of free-fall motion mentioned in the previous section can be examined graphically. **Case 1** is that of an object dropped off a cliff. Figure 2.22 shows the relationship between a velocity–time graph **(b)** and its corresponding acceleration–time graph **(a)** for this type of free-fall motion. The downward direction is negative.

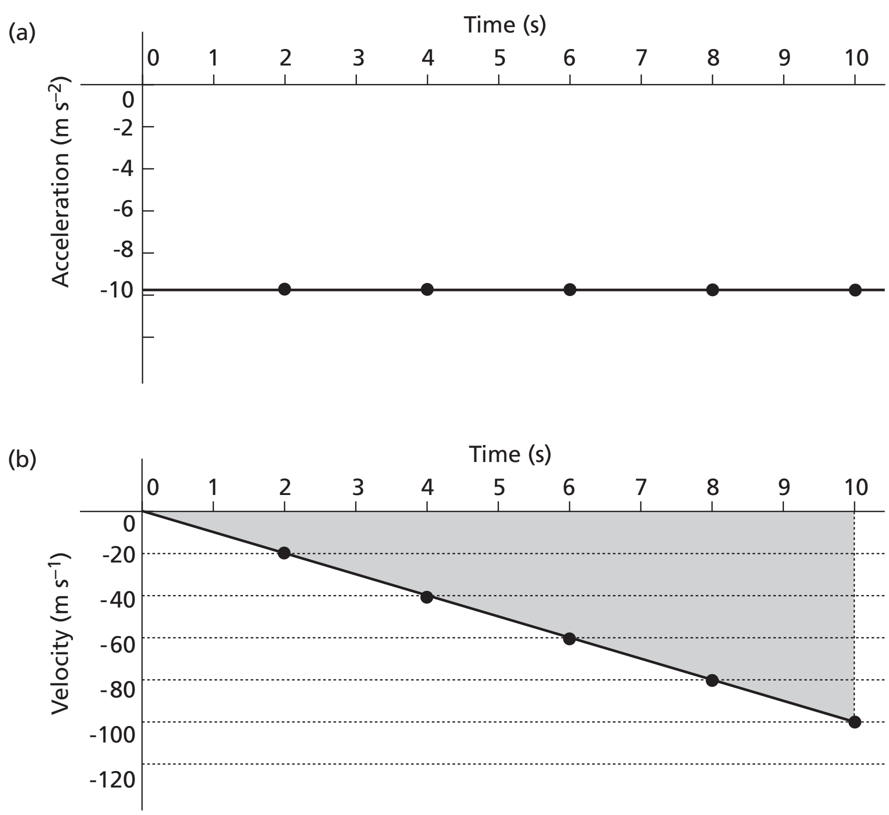


Figure 2.22
(a) An acceleration–time graph; **(b)** a velocity–time graph. (The shaded area indicates the displacement.)

Case 2 is that of an object thrown upward into the air and allowed to return to its starting place. Figure 2.23 shows the graphs of motion of a ball thrown in this manner. Note that the acceleration is constant, even at the top of flight when the ball is stationary. Again, down is negative.

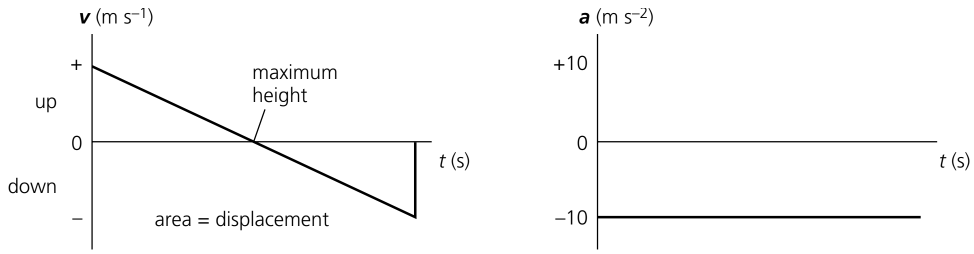


Figure 2.23

Questions

30 What does the slope of the line in the ***v***–*t* graph of Case 1 (Figure 2.22) represent?

31 Which one of the graphs in Figure 2.24 is the displacement–time graph of the rock's motion in Case 2 (Figure 2.23)?

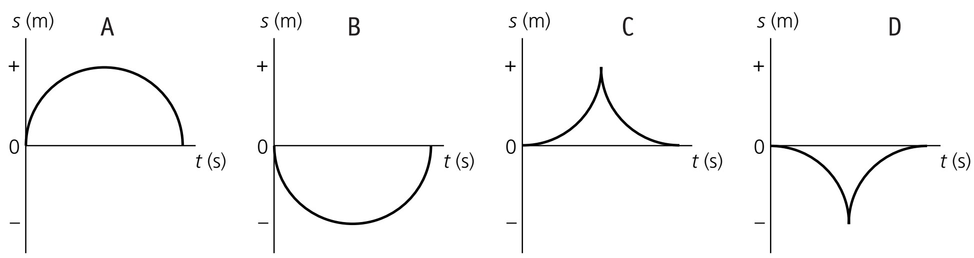


Figure 2.24

32 Draw a displacement–time graph of the motion of the ball as described in Case 1 (Figure 2.22).

2.12 MEASURING MOTION

Motion of an object can be recorded by using a **ticker timer** as shown in Photo 2.3. It has been specifically designed for physics experiments and has little other use outside the physics laboratory. It consists of a pointed hammer, which vibrates up and down 50 times per second. When a paper tape is pulled through the timer, a piece of carbon paper allows an imprint of the hammer to be made on the paper. The distance between successive dots can be used to calculate the velocity of the moving object as the time interval is a constant $\frac{1}{50}$ of a second (0.02 second).

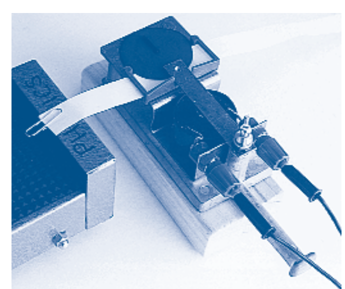


Photo 2.3
A ticker timer.

Consider a section of tape as shown in Figure 2.25. Table 2.12 lists the data from the tape.

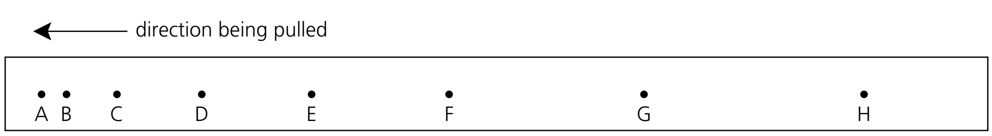


Figure 2.25
A segment of ticker timer tape.

Table 2.12

DOT	A	B	C	D	E	F	G	H
t (seconds)	0	0.02	0.04	0.06	0.08	0.10	0.12	0.14
s (cm)	0	0.3	1.1	2.6	4.6	7.1	10.3	14.0
v (cm/s)	0	27.5	57.5	87.5	112.5	142.5	172.5	–

The average velocity can be determined by dividing the total displacement (14.0 cm) by the time elapsed (0.14 s) to give 100 cm s^{-1}. The instantaneous velocity at each dot can be calculated by measuring the distance travelled between dots either side of the one being considered. For example, to calculate the velocity at dot D, the distance between dots C and E is measured (3.5 cm — see Figure 2.26) and this is divided by the time interval (2×0.02 s). The velocity at D is thus 87.5 cm s^{-1}. The velocity of the other dots can also be calculated.

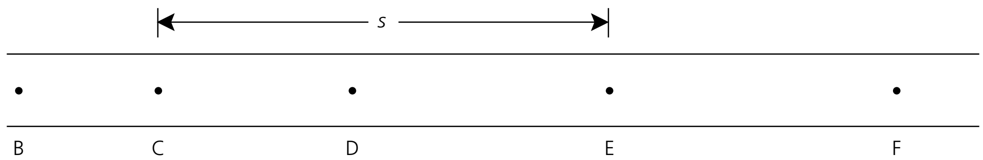


Figure 2.26
Tape.

If acceleration is constant, a graph of velocity vs time should be linear and the slope of this line will equal the acceleration.

To calculate the acceleration at a point, the velocity at the dot before this point and at the dot after the point should be determined. The difference ($\boldsymbol{v} - \boldsymbol{u}$), when divided by the time elapsed, will equal the acceleration.

For example, the acceleration at point D can be calculated by subtracting the velocity at C from the velocity at E and dividing by 0.04 seconds: $\boldsymbol{v}_C = 57.5$ cm s^{-1}, $\boldsymbol{v}_E = 112.5$ cm s^{-1}, hence $\Delta\boldsymbol{v} = \boldsymbol{v}_E - \boldsymbol{v}_C = 55$ cm s^{-1}. The result: $\boldsymbol{a}_D = \frac{\Delta\boldsymbol{v}}{t} = \frac{55}{0.04} = 1375$ cm s^{-2} is the acceleration at D. The acceleration at E likewise is 1375 cm s^{-2}. You should check this for yourself.

Questions

33 The following questions refer to the section of tape described above. (Figure 2.26)

(a) Plot the displacement vs time graph of the data above.
(b) Calculate the slope of the graph at points C and F. How do these slopes compare with the calculated velocity at these points?
(c) Plot the velocity vs time graph of the above data.
(d) Calculate the area under the line to point G. How does it compare with the displacement at G (10.3 cm)?
(e) Calculate the slope of the velocity vs time graph. How does it compare with the calculated acceleration (1375 cm s^{-2})?

34 The following questions refer to the ticker tape shown in Figure 2.27.

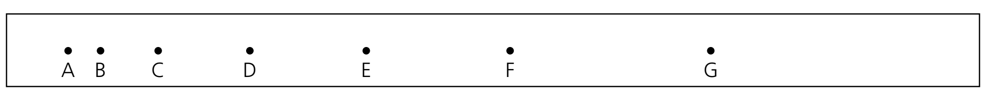


Figure 2.27
For question 34.

(a) Draw up a data table similar to Table 2.10 and measure the displacements using your ruler. Enter the displacements in your data table. Do not write in this book.
(b) Plot a displacement–time graph.
(c) Calculate the slope of the tangent at point E.
(d) Calculate the velocity at each dot and add to the data table.
(e) How does the velocity at E compare with the slope at E on the *s*–*t* graph?
(f) Plot a velocity–time graph and draw a line of best fit.
(g) Calculate the slope of the line.
(h) Calculate the acceleration at points B, C, D and E and add these to the data table.
(i) How does this compare with the slope of the $\boldsymbol{v}$–t graph?
(j) Calculate the displacement at point F by measuring the area under the $\boldsymbol{v}$–t graph. How does it compare with the actual displacement at F as measured on the tape?

2.13 ELECTRONIC RECORDING AND COMPUTER INTERFACING

There are several devices that enable motion to be recorded electronically. Data-loggers are used extensively in research and industry to monitor the performance of various devices under test. The data-logging system comes with a package including an interface system and various sensors to pick up data from the environment such as motion, temperature, voltage, sound and light. The sensor converts physical or chemical changes into electrical signals; these **analog** signals are carried to the interface system where the signals are digitised. Such **digital** signals can be analysed and displayed on the computer monitor and calculations can be performed. The graphical display function that accompanies data-logging programs transforms data into graphs, which help show trends and anomalies.

Data-loggers are used for measuring not only motion but also an enormous range of other data. You will have heard of the heart monitors in hospitals and 'black box' flight recorders in planes. But they are also used for purposes as diverse as designing and producing torpedos, counting biscuits on a production line, measuring causes of stress on individual soldiers in combat situations, and monitoring the drying of paint and curing in industrial ovens. Data-logging equipment is in use at smelters, refineries, tailings dams, mines, landfills, construction sites, manufacturing and processing plants, and industrial and hazardous waste sites; and meteorological conditions can be monitored to yield data for determining air stability or for use in air quality and dispersion modelling.

Activity 2.5 DATA-LOGGER IN MOTORSPORT

Try the following as a good stimulus response task, or it could be the start of a non-experimental investigation.

Racing cars

An interesting use of data-logging is in the racing car industry. Car manufacturers need to run their cars at high speeds for predetermined times as part of their endurance testing program, so they are packed with temperature and pressure sensors that feed data into computers.

The success of a racing car depends on hundreds of components working together at peak performance under the most extreme conditions. Components such as displacement sensors are designed to control and monitor a growing number of vital functions on racing cars and supply information to engineers, who can then help trim precious seconds off the car's lap times. Although most categories of motor racing do not allow the performance of the suspension to be modified during a race, the use of computerised data-logging in testing and practice allows race engineers to tune the suspension to match the particular conditions and type of circuit.

Monitoring the movement of the suspension with displacement sensors allows electrical signals to feed back to the logging/telemetry system and then display a graphical representation of the car's performance around a track. Using the data, engineers can easily recognise areas where improvements can be made, and fine-tune the car by adjusting ride heights and stiffness to suit a particular track and driver. Movement of the suspension can usually be sensed by a linear displacement sensor, but some need rotary sensors.

Throttle controls have a rotary motion, so a rotary displacement transducer (sensor) can be attached to the linkage. The position of the throttle mechanism is usually in a very hostile environment such as the top of the engine or underneath air intake ducts, so either device must be extremely rugged and able to withstand high levels of shock, vibration and high temperatures.

Photo 2.4
Formula One racing cars have a huge number of transducers being monitored by data-loggers to give them a winning edge. Shown here is world champion Michael Schumacher in his Ferrari F2002, winning the French Grand Prix.

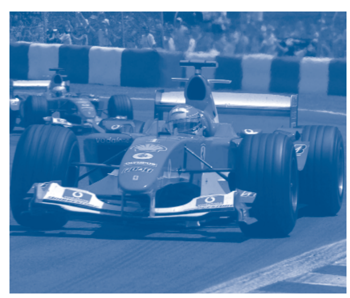
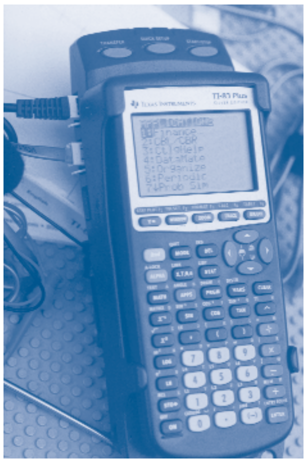


Photo 2.5

Many calculator manufacturers - including Texas Instruments and Casio - make attachable data loggers. In this photo a TI-CBL2 computer based laboratory (data logger) is connected to a TI-83 graphing calculator. The CBL/CBR program shown on the displat provides the interface for this to work. A huge range of probes are available to connect to these devices.



Special 'paddles' on the driver's steering wheel electronically control the clutch actuating mechanism on today's high-performance racing cars, overcoming the need for the driver to use the feet to engage or disengage the clutch. This arrangement allows faster up-changing and down-changing of the gears during acceleration and braking.

When it comes to braking, recent developments in GT and Formula One brake caliper design have enabled systems to be fitted to monitor the wear of the brake pads and discs during a race. Advising the driver to back off by one second a lap can make a significant difference to brake wear. The movement of the brake caliper piston is sensed by a very small sensor embedded in the caliper body, which has been specially designed to withstand extremes of shock and vibration from the track as well as the high temperatures from the brake discs. The back of the brake pads can reach temperatures as high as 400°C, while the caliper body can reach 150–200°C. On Formula One cars up to eight sensors per car are fitted. The signals from the sensor are fed to the car's data-acquisition system and can tell race engineers the condition of the brake pad and disc wear characteristics.

Question: As a work experience student you have been asked to prepare a leaflet for some Year 8 students who will be visiting the racing car development laboratories of the Ford Motor Company. In 200 words what would you say?

Questions

35 A car moving at 30 m s^{-1} decelerates at a uniform rate of 1.5 m s^{-2}. How many seconds will it take to stop and how far will it travel in this time?

36 Analysis of traffic camera data shows that a car 4 m long takes 1.2 seconds to cross an intersection 16 m wide. The time taken is from the moment the car's headlights enter the intersection to the moment the tail-lights depart. Was the car exceeding the speed limit of 60 km h^{-1}?

Practice questions

The relative difficulty of these questions is indicated by the number of stars beside each question number: * = low; ** = medium; *** = high.

Review — applying principles and problem solving

***37** To help you rearrange equations and substitute numbers, do these simple calculations (Table 2.13):

Table 2.13

	DISPLACEMENT	TIME	VELOCITY
(a)	300 m	6 s	
(b)	150 km	4 h 30 min	
(c)		30 s	340 m/s
(d)		3 h 15 min	220 km/h
(e)	300 m		15 m/s
(f)	3.5×10^6 km		65 km/h

***38** The best time by an Australian in the 40 km marathon is that of Robert de Castella, who ran the Boston Marathon in 1986 in 2 h 7 min 51 s. Calculate: **(a)** his average speed; **(b)** the time it would take Michael Johnson if he ran the distance at 10.15 m s^{-1}.

***39** The fastest lap of the British Motorcycle Grand Prix at Donnington Park was in 1993 by Luca Cadalora on a 500 cc Yamaha in 1 min 34.716 s, averaging 152.908 km/h. **(a)** How long is the track? **(b)** If the race was 30 laps and he took 47 min 45.630 s, what was his average speed for the race?

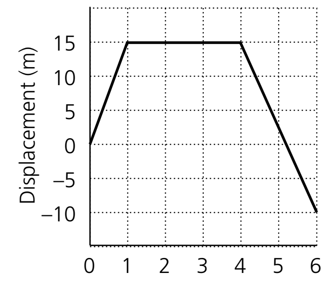
***40** Australian fast bowler Brett Lee was electronically timed to deliver a cricket ball at 157.4 km h^{-1} in the second test against South Africa in 2002. How many seconds would it take for the ball to travel the 20 m length of the cricket pitch?

***41** A person runs in a straight line 84 m south in 9.0 s and then 160 m north in 18.0 s. What is his **(a)** displacement; **(b)** average speed; **(c)** average velocity?

***42** A car travels on a straight road for 50 km at 30 km h^{-1}. It then continues in the same direction for another 20 km at 60 km/h. What is the average velocity of the car during this trip?

***43** The graph in Figure 2.28 shows the displacement of a radio-controlled car being driven in a straight line:
- **(a)** What is its displacement after 3 s?
- **(b)** Calculate how far it travelled in the 6 s.
- **(c)** At what stage was its velocity the greatest?
- **(d)** When was it stationary?
- **(e)** When was its velocity constant but not zero?
- **(f)** What was its velocity at 5 s?

Figure 2.28
For question 43.



***44** Practise applying the acceleration formula by completing Table 2.14. Do not write in this book.

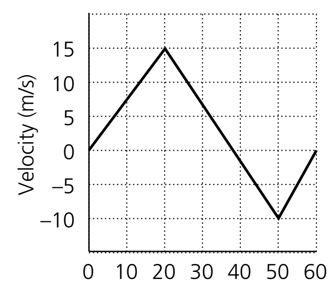
Table 2.14

	v (m s^{-1})	u (m s^{-1})	t (s)	a (m s^{-2})
(a)	100	40	3.5	?
(b)	60	130	0.85	?
(c)	250	?	1.5	4.0
(d)	16.7	27.0	?	−1.5
(e)	0.55	0.15	?	60
(f)	?	0	1.2×10^{-3}	2.0

***45** A car with an initial velocity of 3.0 m s^{-1} has a velocity of 34 m s^{-1} after 3.0 s. Calculate **(a)** its acceleration; **(b)** its average velocity; **(c)** how far it moved in its third second of motion; **(d)** its speed after travelling 20 m.

***46** The graph in Figure 2.29 shows the motion of a girl on rollerblades as a function of time.
- **(a)** Calculate her displacement after 50 seconds.
- **(b)** Calculate the distance she travelled in the minute.
- **(c)** At what stage was her acceleration the greatest?
- **(d)** When was she stationary?
- **(e)** When was her velocity constant but not zero?

Figure 2.29
For question 46.



***47** Table 2.15 will give you practice in selecting equations of motion and substituting values into them. Complete the table but do not write in this book.

Table 2.15

QUESTION	s (m)	u (m s^{-1})	v (m s^{-1})	a (m s^{-2})	t (s)
(a)		0		3	1.5
(b)	200	0			1.4
(c)		20	65	2.6	
(d)	315	7.5		2.5	
(e)	400	25		−0.4	
(f)		30	8.7		2.3
(g)	1550	80			800

48 A cyclist is travelling at a constant 10 m s^{-1} when he begins to coast up a hill. Assuming that he decelerates uniformly at 1.8 m s^{-2}, calculate **(a) how far he will travel before coming to rest; **(b)** how long this will take.

49 A pedestrian steps on to the road while an approaching car is travelling at 30 km h^{-1}. If the driver's reaction time is 0.3 s and the braking deceleration is 4.5 m s^{-1}, calculate **(a) the stopping distance; **(b)** the stopping time.

**50 A car travelling at 100 km h^{-1} takes 65 m to stop after the driver sees a child run on to the road chasing a ball. If the driver's reaction time is 0.25 s, calculate the deceleration of the car.

*51 In the 1993 British Motorcycle Grand Prix, Kevin Schwantes was eliminated after a crash. *Australian Motorcycle News* described the crash: 'Schwantes was the first to crash after asking too much of a cold rear tyre. He hit the grass at 290 km/h and slid to a halt in a set of sand traps 50 m down the track.' Calculate Schwantes' deceleration in this accident.

*52 The results of experiments published in 1966 show that nerve impulses can travel at 288 km/h in the human body. How many seconds would elapse if they travelled at this speed from your toe to your brain (say 170 cm)? Assume the speed is constant.

*53 The Lee Enfield Rifle (.303) was used by Commonwealth Forces during the Second World War. Its projectiles had a muzzle velocity of 745 m s^{-1} and came to rest at a range of 700 m. Calculate **(a)** the deceleration (assumed uniform); **(b)** the time of flight.

*54 Suppose a rocketship in deep space moves with a constant acceleration of 9.8 m s^{-2}, which will give the illusion of normal gravity during the flight. **(a)** If it starts from rest, what time will it take to reach a speed one-tenth that of the speed of light (3×10^8 m s^{-1})? **(b)** How far will it travel in doing so?

55 Consider a case where air resistance is taken into account. A tennis ball was dropped from a 120 m high cliff and accelerated uniformly to a terminal speed of 20 m s^{-1} after 5 s. From then on to the ground it travelled at this speed. Calculate **(a) its acceleration over the first 5 s; **(b)** how far it travelled before it reached terminal speed; **(c)** its total time of flight; **(d)** its impact velocity; **(e)** its average velocity for the entire flight.

**56 Consider cases where an object is thrown vertically into the air. In these cases upward is still the positive direction. 'Time of flight' means the total time from launch to impact. Complete Table 2.16:

Table 2.16

	INITIAL VELOCITY u (m/s)	TIME OF FLIGHT t (s)	MAXIMUM HEIGHT s (m)
(a)	10		
(b)			100
(c)		5.5	

57 The single cable supporting a construction elevator breaks when the elevator passes the sixth floor (25 m) on its way up while at a speed of 3.0 m s^{-1}. **(a) Calculate velocity on impact. **(b)** How much time will elapse before the elevator strikes the ground?

**58 In an experiment to investigate the relationship between time, displacement, velocity and acceleration, a trolley was allowed to run down an inclined plane with its motion being recorded by a ticker timer. Figure 2.30 shows a 1 m length of the tape cut into five continuous segments so that it can be displayed in this textbook. Note the time interval between successive dots is 0.02 second.

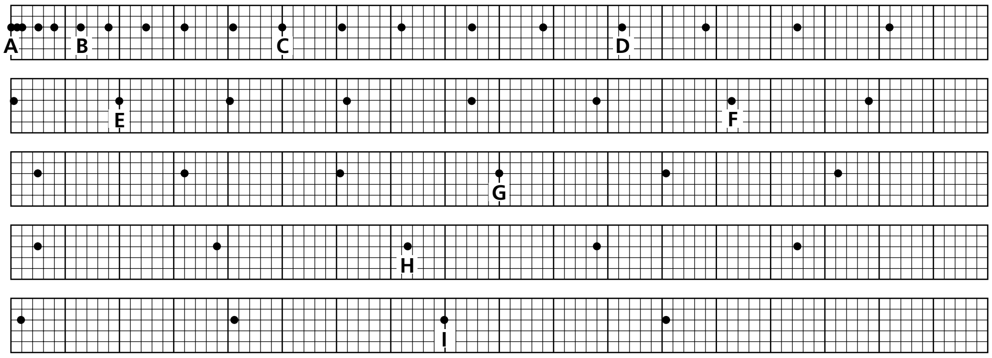


Figure 2.30
A ticker timer tape cut into five segments to fit the page.

Part A Displacement

(a) Every fifth dot was marked alphabetically. Check that this is correct.
(b) Calculate the time elapsed for each lettered dot and measure the displacement of each dot from the start. Add this to the data table (Table 2.17). Do not write in this book. Redraw the data tables or photocopy them.

Table 2.17

DOT	A	B	C	D	E	F	G	H
t (seconds)	0	0.1	0.2	0.3				
s (cm)								

(c) Plot a graph of *t* (*x*-axis) versus *s* (*y*-axis).
(d) What is the displacement of the last lettered dot (I)?

Part B Velocity

(e) Draw tangents at each of the lettered dots C, E and G and calculate their slope. Add to Table 2.18 in the second row ('slope').

Table 2.18

DOT	A	B	C	D	E	F	G	H
t (seconds)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7
Slope (cm/s)								
v (cm/s)	0							

(f) Calculate the velocity of each lettered dot by measuring the distance between dots either side of each lettered dot and dividing by the time interval (2×0.02 s). Add these data to Table 2.19. in the ***v*** row.
(g) How does the average velocity for each lettered dot compare with the instantaneous velocity as calculated from the slope of the *s*–*t* graph in Question **(c)**?
(h) For the average velocity as calculated in question **(g)** plot velocity vs time (*x*-axis) and draw a line of best fit.
(i) Determine the area under the graph up to the last lettered dot (I). How does this compare with the measured displacement of dot I?

Part C Acceleration

(j) Calculate acceleration by measuring the slope of the graph of ***v***–*t*.
(k) Calculate acceleration from the tape for dots C, E and G by subtracting the velocity five dots before from the velocity five dots after and dividing by the time interval over the ten dots. For example, to calculate the velocity at C, subtract the velocity at B from the velocity at D and divide by ten dot intervals of time. Add this to Table 2.19.

Table 2.19

DOT	A	B	C	D	E	F	G	H	I
t (seconds)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
v_1 (cm s^{-1})									
v_2 (cm s^{-1})									
a (cm s^{-2})									

(l) Plot a graph of acceleration vs time and draw a line of best fit.
(m) Calculate the average acceleration by averaging the acceleration at the lettered dots C, E and G.
(n) How does the value of average acceleration compare with the slope of the v–t graph?

Note: if you have access to a computer and spreadsheet, you may like to set up the spreadsheet to make the various calculations.

****59** Table 2.20 is taken from a *Wheels Magazine* comparison of some popular four-cylinder cars.

Table 2.20

	MAZDA 626	SUBARU LIBERTY	TOYOTA CAMRY
Engine capacity (litres)	1.991	2.212	2.164
Engine — Max. power (kW)	85	100	95
Top speed (km/h):			
• First gear	63	62	63
• Second gear	113	118	113
• Third gear	174	175	176
• Fourth gear	190	195	185
Acceleration (seconds):			
• 0–60 km/h	5.7	5.2	5.8
• 0–80 km/h	9.3	8.3	9.3
• 0–100 km/h	13.6	12.0	13.6
• 0–120 km/h	20.1	17.5	20.1
Standing 400 m (km/h):	19.1 (117)	18.3 (123)	19.1 (118)
• 40–70 km/h	4.1	3.5	3.9
• 60–90 km/h	5.5	4.9	5.5
• 80–100 km/h	7.2	6.0	7.1
• 100–130 km/h	10.6	9.0	11.0

(a) Which car has the best overall acceleration? Justify your choice.
(b) Which, if any, of the cars reaches its maximum speed in less than 400 m?
(c) Does a car's ability to accelerate get progressively less at higher speeds? Justify your answer.
(d) Calculate the distance over which the 40–70 km h^{-1} acceleration test would have occurred for the Toyota.
(e) List five other criteria that would be important to include in a car comparison.
(f) 'The greater the engine power, the greater the acceleration.' Comment critically on this claim with reference to the above data.
(g) 'The greater the engine capacity the greater the acceleration.' Comment critically.

Extension — complex, challenging and novel

***60 Chris beats Sandy by 10 m in a 100 m sprint. Chris, wanting to give Sandy an equal chance, agrees to race her again but to begin 10 m behind the starting line. Does this really give Sandy an equal chance?

***61 The General Dynamics F-111 jet has been in service with the RAAF since 1963. Its maximum speed above 50 000 feet is 825 m s^{-1} (Mach 2.5) but this drops to 396 m s^{-1} (Mach 1.2) at sea level because of air resistance. Calculate the deceleration as an F-111 drops and decreases speed as stated in 30 s.
Note: Mach numbers are the number of times the speed is greater than the speed of sound at that place (330 m s^{-1}).

***62 Two bus stops are 1200 m apart. A bus accelerates at 0.95 m s^{-2} from rest through the first quarter of the distance and then travels at constant speed for the next two quarters and decelerates to rest over the final quarter.
(a) What was the maximum speed? **(b)** What was the total time taken for the journey? **(c)** Draw a $\boldsymbol{v}$–t graph of the journey.

***63 A basketball player, standing near the basket to grab a rebound, jumps 76.0 cm vertically. On his way up, how much time does he spend **(a)** in the bottom 15 cm of his jump; **(b)** in the top 15 cm of his jump? Does this help to explain why such players seem to hang in the air at the tops of their jumps?

***64 A juggler tosses balls vertically into the air. How much higher must they be tossed if they are to spend twice as much time in the air?

***65 A stone is dropped off a bridge 50 m above the water. Exactly 1 s later another stone is thrown down and both stones strike the water together. **(a)** What must the initial speed of the second stone have been? **(b)** Plot a $\boldsymbol{v}$–t graph of both stones on the one graph.

***66 Who would have the more thrilling ride: Kitty O'Neil in her dragster, which reached 628 km/h in 3.72 s or Eli Beeding who reached 116 km/h in 0.04 s on a rocket sled? Justify your choice by commenting on what determines how thrilling a ride might be — the speed, the time, the acceleration or something else.

***67 A person standing on the edge of a cliff throws a ball straight up with speed '$\boldsymbol{u}$', allowing it to crash on to the rocks below. He later throws a ball with the same speed '$\boldsymbol{u}$' straight down. Which ball has the higher speed when it hits the rocks? Neglect air resistance.

***68 A ball is dropped down an elevator shaft and then 1 s later a second ball is dropped. **(a)** How does the distance between the two balls vary as time passes? **(b)** How does the ratio $\boldsymbol{v}_1 : \boldsymbol{v}_2$ vary with time?

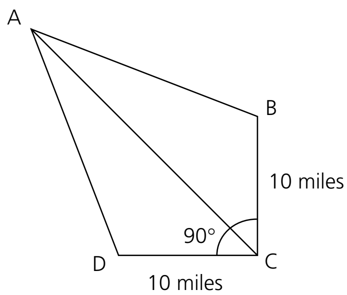
***69 The Australia vs USA Nitro-Harley Challenge is the world's richest motorcycle drag race meeting. One of the most successful riders, Phil Hill (USA), is 61 years old. With a 103 cubic inch nitromethane injected 350 horsepower engine he can cover the standing quarter mile (400 m) in 7.22 seconds with a final speed of 305 km h^{-1}. The acceleration required to cover 400 m from a standing start in 7.22 s is more than the acceleration needed to reach 305 km h^{-1} from a standing start in the same time. How can you explain this apparent discrepancy in the calculations?

***70 A rule-of-thumb in motorcycle drag racing is that 'sixty pounds is three-tenths of a second'. This is meant to show how a rider's weight affects the time to cover 400 m from a standing start. Australian national record holder Bill Curry has a best time of 6.92 s. Calculate how much his average acceleration would be if he was 20 kg heavier.
Note: 1 kg equals 2.2 pounds.

***71 The speed of non-land-based vehicles such as ships and planes is usually measured in 'knots'. A knot is one nautical mile (6080 feet) per hour. To measure the speed of a ship, a line with knots at set intervals was attached to a log that was thrown overboard from the stern of a ship. As the log drifted away from the ship a sailor would count how many knots passed through his fingers while the sandglass emptied. Usually the 'log line' had knots every 100 feet and the sandglass emptied in 1 minute. **(a)** Prove that a speed of 30 knots equals 30 nautical miles per hour. **(b)** How many km h^{-1} is 30 knots if 1 foot equals 0.305 metres?

***72 If you have access to a computer, set up a spreadsheet to compute the distance an object falls, as a function of time of falling, near the surface of the Earth ($\boldsymbol{g}$ = 9.8 m s^{-2}); our Moon ($\boldsymbol{g}$ = 1.6 m s^{-2}); Mars ($\boldsymbol{g}$ = 3.8 m s^{-2}) and the Sun ($\boldsymbol{g}$ = 270 m s^{-2}). Compute the distance of fall for each fifth of a second from 0 to 2 seconds.

Figure 2.31
The 37-year puzzle.



***73 The Incredible Tale of the 37-year Puzzle. This puzzle remained unsolved for 37 years until *Popular Science Magazine* published it again in October 1976. Two thousand responses were sent in and five different solutions appeared. The problem (Figure 2.31): A man always drives at the same speed. He makes it from A direct to C in 30 minutes; from A through B to C in 35 minutes; and from A through D to C in 40 minutes. How fast does he drive?

***74 Galileo's first attempt at producing a law of falling bodies was limited by his lack of mathematical means of describing continuously varying motion. In a letter to a friend Paolo Scarpi in 1604 he wrote: 'Spaces traversed in natural motion are in squared proportion of the times, and consequently the spaces traversed in equal times are as the odd numbers beginning with unity. And the principal in this, that the naturally moving body increases its velocity in the proportion that it is distant from the origin of the motion.' Can you convert this to mathematical statements and then comment on whether Galileo was correct with these early theories?

TEST YOUR UNDERSTANDING

(Answer true or false)
- Two objects side by side must have the same speed.
- Acceleration is in the same direction as velocity.
- Velocity is a force.
- Heavier objects fall just a bit faster than light ones.
- If velocity is zero, acceleration is zero.
- In the absence of gravity all things move with equal ease.
- At the top of its flight a vertically thrown object has zero acceleration.

***75 The Sukhoi Su-29 is a Russian built two-seat aerobatic competition aircraft becoming popular in Australian competitions. If one was flying at its cruising speed of 160 knots (298 km/h) and an altitude of 1000 m and suddenly encountered terrain sloping upward at 4.3°, an amount difficult to detect, how much time would the pilot have to make a correction if he is to avoid flying into the ground?

***76 An article in the newspaper quoting a safety expert said that: 'An unrestrained child in a 50 km/h car crash suffered the same effects as being dropped on to concrete from a building's second floor. It said some parents still held the belief that merely placing children in the back seat would protect them in a crash.' Confirm or refute these comments made by the paper, making whatever approximations are required.

***77 A car has an oil leak from the sump and a drop falls every 2 seconds. Draw a diagram of how the spots would appear on a 64 m driveway as the car accelerates up it from rest at 2 m s^{-2}. Assume the first drop falls at the instant the car moves.